A New Measure for M/M/1 Queueing System with Non Preemptive Service Priorities

Indu Jindal[1], Shweta Sharma[2]
[1]Post Graduate Govt. College for Girls, Sector 42, Chandigarh
[2]Rayat Bahra College of Engineering and Bio-Technology for Women, Mohali (Punjab)

ABSTRACT

This paper studies M/M/1 queue under non preemptive service priority discipline. The unit with higher priority gets the service first without interrupting the service of non- priority unit if it is already in service. The study obtains the Laplace Transform of explicit time-dependent probabilities of exactly i arrivals, j departures of priority units and k arrivals, l departures of non-priority units by time t by solving the Laplace Transform of difference equations iteratively. By inverting the Laplace Transforms actual probabilities can be known. Few explicit results have been derived which helped in the verification of the model. With the help of these explicit transient state probabilities, other measures of interest can be obtained.

Keywords

Queues, Priority, Transient behaviour.

INTRODUCTION

It was Cobham [3] who considered non-preemptive priority queueing discipline with Poisson input and exponential holding time. Phipps [14] extended the study to the case when the numbers of priorities are continuous. White and Christie [18] obtained the steady state solution of preemptive priority queues with Poisson arrivals and exponential service time. Heathcote [6] found the time-dependent solution of the same model. Later Miller [12], Heathcote [7, 8], Jaiswal [9, 10], Gaver [4], Hawkes [5], Sharda [15-17], Kao & Wilson [11], Choi et.al [2], Balter et.al [1] studied some queueing systems with priority as queue discipline.

In the present paper, the concept of Pegden and Rosenshine [13] is applied to find out the explicit time dependent probabilities for exact number of arrivals and departures of priority units as well as of non-priority units by a given time recursively. The practical situation which corresponds the above problem can be that of a repair shop of electronic items. In winter season, repair of electric heaters will be on priority as compared to other electronic items. The mechanic can know the number of repaired heaters and number of total heaters received for repair and similarly for other electronic items by a given time.

QUEUEING MODEL

The queueing system studied in this paper is described by the following assumptions:

(i) The priority units and non-priority units arrive in a Poisson distribution with parameters λ_1 and λ_2 respectively.

(ii) The service time follows an exponential distribution with parameter μ.

(iii) When there is one priority unit in the system then it may or may not be in channel. Let p and q be the probabilities that the service is being done of priority or non priority units such that $p+q=1$.

(iv) When there are more than one priority units in the system then service will be done of priority unit only.

(v) The stochastic processes involved viz.
  (a) Arrival of units.
  (b) Departure of units;
  are statistically independent.

Define

$P_{i,j,k,l}(t)$ = Probability of i arrival, j departure of priority units and k arrival, l departure of non - priority units by time t.
$i \geq j, k \geq l$

$P_n(t)$ = Probability of n arrivals by time t.

$$= \sum_{j=0}^{i}\sum_{l=0}^{k} P_{i,j,k,l}(t), \text{ where } i + k = n$$

Initially $P_{0,0,0,0,F}(0)=1$

$P_{0,0,0,0,B}(t)=0$

The difference- differential equations governing the system are:

$$\frac{d}{dt}P_{i,j,k,l}(t) = -(\lambda_1 + \lambda_2 + \mu)P_{i,j,k,l}(t) + \lambda_1 P_{i-1,j,k,l}(t) + \lambda_2 P_{i,j,k-1,l}(t) + \mu P_{i,j-1,k,l}(t)$$
$$1 \leq j \leq i-2, 0 \leq l \leq k-1 \quad (1)$$

$$\frac{d}{dt}P_{i,i,k,l}(t) = -(\lambda_1 + \lambda_2 + \mu)P_{i,i,k,l}(t) + \lambda_2 P_{i,i,k-1,l}(t) + \mu p P_{i,i-1,k,l}(t) + \mu P_{i,i,k,l-1}(t)$$
$$i \geq 1, \ 1 \leq l \leq k-1 \quad (2)$$

$$\frac{d}{dt}P_{i,i-1,k,l}(t) = -(\lambda_1 + \lambda_2 + \mu)P_{i,i-1,k,l}(t) + \lambda_1 P_{i-1,i-1,k,l}(t) + \lambda_2 P_{i,i-1,k-1,l}(t) + \mu P_{i,i-2,k,l}(t) + \mu q P_{i,i-1,k,l-1}$$
$$1 \leq j \leq i-2, 0 \leq l \leq k-1 \quad (3)$$

$$\frac{d}{dt}P_{i,0,k,l}(t) = -(\lambda_1 + \lambda_2 + \mu)P_{i,0,k,l}(t) + \lambda_1 P_{i-1,0,k,l}(t) + \lambda_2 P_{i,0,k-1,l}(t)$$
$$i \geq 2, 0 \leq l \leq k-1 \quad (4)$$

$$\frac{d}{dt}P_{1,0,k,l}(t) = -(\lambda_1 + \lambda_2 + \mu)P_{1,0,k,l}(t) + \lambda_1 P_{0,0,k,l}(t) + \lambda_2 P_{1,0,k-1,l}(t) + \mu q P_{1,0,k,l-1}(t)$$
$$1 \leq l \leq k-1 \quad (5)$$

$$\frac{d}{dt}P_{0,0,k,l}(t) = -(\lambda_1 + \lambda_2 + \mu)P_{0,0,k,l}(t) + \lambda_2 P_{0,0,k-1,l}(t) + \mu P_{0,0,k,l-1}(t) \quad 1 \leq l \leq k-1 \quad (6)$$

$$\frac{d}{dt}P_{i,j,k,k}(t) = -(\lambda_1 + \lambda_2 + \mu)P_{i,j,k,k}(t) + \lambda_1 P_{i-1,j,k,k}(t) + \mu P_{i,j-1,k,k}(t)$$

1

$$\leq j \leq i-2, \forall k \quad (7)$$

$$\frac{d}{dt}P_{i,i-1,k,k}(t) = -(\lambda_1 + \lambda_2 + \mu)P_{i,i-1,k,k}(t) + \lambda_1 P_{i-1,i-1,k,k}(t) + \mu P_{i,i-2,k,k}(t) + \mu q P_{i,i-1,k,k-1}(t)$$
$$i \geq 2, k \geq 1 \quad (8)$$

$$\frac{d}{dt}P_{i,i,k,k}(t) = -(\lambda_1 + \lambda_2)P_{i,i,k,k}(t) + \mu P_{i,i-1,k,k}(t) + \mu q P_{i,i,k,k-1}(t) \quad i \geq 1, k \geq 1 \quad (9)$$

$$\frac{d}{dt}P_{i,i,k,0}(t) = -(\lambda_1 + \lambda_2 + \mu)P_{i,i,k,0}(t) + \lambda_2 P_{i,i,k-1,0}(t) + \mu p P_{i,i-1,k,0}(t) \quad i \geq 1, k \geq 1 \quad (10)$$

$$\frac{d}{dt}P_{i,0,k,k}(t) = -(\lambda_1 + \lambda_2 + \mu)P_{i,0,k,k}(t) + \lambda_1 P_{i-1,0,k,k}(t) \quad i \geq 2, \forall k \quad (11)$$

$$\frac{d}{dt}P_{0,0,k,k}(t) = -(\lambda_1 + \lambda_2)P_{0,0,k,k}(t) + \mu P_{0,0,k,k-1}(t) \quad k \geq 1 \quad (12)$$

$$\frac{d}{dt}P_{i,i-1,k,0}(t) = -(\lambda_1 + \lambda_2 + \mu)P_{i,i-1,k,0}(t) + \lambda_1 P_{i-1,i-1,k,0}(t) + \lambda_2 P_{i,i-1,k-1,0}(t) + \mu q P_{i,i-2,k,0}(t)$$
$$i \geq 2, k \geq 1 \quad (13)$$

$$\frac{d}{dt}P_{0,0,k,0}(t) = -(\lambda_1+\lambda_2+\mu)P_{0,0,k,0}(t) + \lambda_2 P_{0,0,k-1,0}(t) \qquad k \geq 1 \qquad (14)$$

$$\frac{d}{dt}P_{i,i,0,0}(t) = -(\lambda_1+\lambda_2)P_{i,i,0,0}(t) + \mu P_{i,i-1,0,0}(t) \qquad i \geq 1 \qquad (15)$$

$$\frac{d}{dt}P_{i,i-1,0,0}(t) = -(\lambda_1+\lambda_2+\mu)P_{i,i-1,0,0}(t) + \lambda_1 P_{i-1,i-1,0,0}(t) + \mu P_{i,i-2,0,0}(t) \qquad i \geq 2 \qquad (16)$$

$$\frac{d}{dt}P_{1,0,k,k}(t) = -(\lambda_1+\lambda_2+\mu)P_{1,0,k,k}(t) + \lambda_1 P_{0,0,k,k}(t) + \mu q P_{1,0,k,k-1}(t) \qquad k \geq 1 \qquad (17)$$

$$\frac{d}{dt}P_{1,0,k,0}(t) = -(\lambda_1+\lambda_2+\mu)P_{1,0,k,0}(t) + \lambda_1 P_{0,0,k,0}(t) + \lambda_2 P_{1,0,k-1,0}(t) \qquad k \geq 1 \qquad (18)$$

$$\frac{d}{dt}P_{1,0,0,0}(t) = -(\lambda_1+\lambda_2+\mu)P_{1,0,0,0}(t) + \lambda_1 P_{0,0,0,0}(t) \qquad (19)$$

$$\frac{d}{dt}P_{0,0,0,0}(t) = -(\lambda_1+\lambda_2)P_{0,0,0,0}(t) \qquad (20)$$

Taking Laplace transform of $P_{i,j,k,l}(t)$ given by $\overline{P}_{i,j,k,l}(s) = \int_0^\infty e^{-st}P_{i,j,k,l}(t)\ dt$, where Re s > 0, equations (1) to (20) transforms to equations (21) to (40).

$$(s+\lambda_1+\lambda_2+\mu)\overline{P}_{i,j,k,l}(s) = \lambda_1\overline{P}_{i-1,j,k,l}(s) + \lambda_2\overline{P}_{i,j,k-1,l}(s) + \mu\overline{P}_{i,j-1,k,l}(s) \qquad 1 \leq j \leq i-2, 0 \leq l \leq k-1 \qquad (21)$$

$$(s+\lambda_1+\lambda_2+\mu)\overline{P}_{i,i,k,l}(s) = \lambda_2\overline{P}_{i,i,k-1,l}(s) + \mu p\overline{P}_{i,i-1,k,l}(s) + \mu\overline{P}_{i,i,k,l-1}(s) \qquad i \geq 1, 1 \leq l \leq k-1 \qquad (22)$$

$$(s+\lambda_1+\lambda_2+\mu)\overline{P}_{i,i-1,k,l}(s) = \lambda_1\overline{P}_{i-1,i-1,k,l}(s) + \lambda_2\overline{P}_{i,i-1,k-1,l}(s) + \mu\overline{P}_{i,i-2,k,l}(s) + \mu q\overline{P}_{i,i-1,k,l-1}(s) \qquad i \geq 2, 1 \leq l \leq k-1 \qquad (23)$$

$$(s+\lambda_1+\lambda_2+\mu)\overline{P}_{i,0,k,l}(s) = \lambda_1\overline{P}_{i-1,0,k,l}(s) + \lambda_2\overline{P}_{i,0,k-1,l}(s) \qquad i \geq 2, 0 \leq l \leq k-1 \qquad (24)$$

$$(s+\lambda_1+\lambda_2+\mu)\overline{P}_{1,0,k,l}(s) = \lambda_1\overline{P}_{0,0,k,l}(s) + \lambda_2\overline{P}_{1,0,k-1,l}(s) + \mu q\overline{P}_{1,0,k,l-1}(s) \qquad l \leq k-1 \qquad (25)$$

$$(s+\lambda_1+\lambda_2+\mu)\overline{P}_{0,0,k,l}(s) = \lambda_2\overline{P}_{0,0,k-1,l}(s) + \mu\overline{P}_{0,0,k,l-1}(s) \qquad 1 \leq l \leq k-1 \qquad (26)$$

$$(s+\lambda_1+\lambda_2+\mu)\overline{P}_{i,j,k,k}(s) = \lambda_1\overline{P}_{i-1,j,k,k}(s) + \mu\overline{P}_{i,j-1,k,k}(s) \qquad 1 \leq j \leq i-2, \forall k \qquad (27)$$

$$(s+\lambda_1+\lambda_2+\mu)\overline{P}_{i,i-1,k,k}(s) = \lambda_1\overline{P}_{i-1,i-1,k,k}(s) + \mu\overline{P}_{i,i-2,k,k}(s) + \mu q\overline{P}_{i,i-1,k,k-1}(s) \qquad i \geq 2, k \geq 1 \qquad (28)$$

$$(s+\lambda_1+\lambda_2+\mu)\overline{P}_{i,i,k,k}(s) = \mu\overline{P}_{i,i-1,k,k}(s) + \mu\overline{P}_{i,i,k,k-1}(s) \qquad i \geq 1, k \geq 1 \qquad (29)$$

$$(s+\lambda_1+\lambda_2+\mu)\overline{P}_{i,0,k,k}(s) = \lambda_1\overline{P}_{i-1,0,k,k}(s) \qquad i \geq 2, \forall k \qquad (30)$$

$$(s+\lambda_1+\lambda_2)\overline{P}_{0,0,k,k}(s) = \mu\overline{P}_{0,0,k,k-1}(s) \qquad k \geq 1 \qquad (31)$$

$$(s+\lambda_1+\lambda_2+\mu)\overline{P}_{i,i-1,k,0}(s) = \lambda_1\overline{P}_{i-1,i-1,k,0}(s) + \lambda_2\overline{P}_{i,i-1,k-1,0}(s) + \mu\overline{P}_{i,i-2,k,0}(s)$$

$$i \geq 2, k \geq 1 \qquad (32)$$

$$(s+\lambda_1+\lambda_2+\mu)\overline{P}_{i,i,k,0}(s) = \lambda_2\overline{P}_{i,i,k-1,0}(s) + \mu p\overline{P}_{i,i-1,k,0}(s) \qquad i \geq 1, k \geq 1 \qquad (33)$$

$$(s+\lambda_1+\lambda_2+\mu)\overline{P}_{0,0,k,0}(s) = \lambda_2\overline{P}_{0,0,k-1,0}(s) \qquad k \geq 1 \qquad (34)$$

$$(s+\lambda_1+\lambda_2)\overline{P}_{i,i,0,0}(s) = \mu\overline{P}_{i,i-1,0,0}(s) \qquad i \geq 1 \qquad (35)$$

$$(s+\lambda_1+\lambda_2+\mu)\overline{P}_{i,i-1,0,0}(s) = \lambda_1\overline{P}_{i-1,i-1,0,0}(s) + \mu\overline{P}_{i,i-2,0,0}(s) \qquad i \geq 2 \qquad (36)$$

$$(s+\lambda_1+\lambda_2+\mu)\overline{P}_{1,0,k,k}(s) = \lambda_1\overline{P}_{0,0,k,k}(s) + \mu q\overline{P}_{1,0,k,k-1}(s) \qquad k \geq 1 \qquad (37)$$

$$(s+\lambda_1+\lambda_2+\mu)\overline{P}_{1,0,k,0}(s) = \lambda_1\overline{P}_{0,0,k,0}(s) + \lambda_2\overline{P}_{1,0,k-1,0}(s) \qquad k \geq 1 \qquad (38)$$

$$(s+\lambda_1+\lambda_2+\mu)\overline{P}_{1,0,0,0}(s) = \lambda_1\overline{P}_{0,0,0,0}(s) \qquad (39)$$

$$(s+\lambda_1+\lambda_2)\overline{P}_{0,0,0,0}(s) = 1 \qquad (40)$$

Solving (21) to (40) iteratively Laplace transform of all the probabilities $P_{i,j,k,l}(t)$ for all values of i, j, k and l can be known and by using inverse Laplace transform $P_{i,j,k,l}(t)$ can be completely known.

FEW EXPLICIT RESULTS

$$\overline{P}_{0,0,0,0}(s) = \frac{1}{s+\lambda_1+\lambda_2}$$

$$\overline{P}_{1,0,0,0}(s) = \frac{\lambda_1}{(s+\lambda_1+\lambda_2)(s+\lambda_1+\lambda_2+\mu)}$$

$$\overline{P}_{0,0,1,0}(s) = \frac{\lambda_2}{(s+\lambda_1+\lambda_2)(s+\lambda_1+\lambda_2+\mu)}$$

$$\overline{P}_{1,1,0,0}(s) = \frac{\mu\lambda_1}{(s+\lambda_1+\lambda_2)^2(s+\lambda_1+\lambda_2+\mu)}$$

$$\overline{P}_{0,0,1,1}(s) = \frac{\mu\lambda_2}{(s+\lambda_1+\lambda_2)^2(s+\lambda_1+\lambda_2+\mu)}$$

$$\overline{P}_{2,0,0,0}(s) = \frac{{\lambda_1}^2}{(s+\lambda_1+\lambda_2)(s+\lambda_1+\lambda_2+\mu)^2}$$

$$\overline{P}_{2,1,0,0}(s) = \frac{{\lambda_1}^2\mu}{(s+\lambda_1+\lambda_2)(s+\lambda_1+\lambda_2+\mu)^2}\left[\frac{1}{s+\lambda_1+\lambda_2} + \frac{1}{s+\lambda_1+\lambda_2+\mu}\right]$$

$$\overline{P}_{2,2,0,0}(s) = \frac{{\lambda_1}^2\mu^2}{(s+\lambda_1+\lambda_2)^2(s+\lambda_1+\lambda_2+\mu)^2}\left[\frac{1}{s+\lambda_1+\lambda_2} + \frac{1}{s+\lambda_1+\lambda_2+\mu}\right]$$

$$\overline{P}_{1,0,1,0}(s) = \frac{2\lambda_1\lambda_2}{(s+\lambda_1+\lambda_2)(s+\lambda_1+\lambda_2+\mu)^2}$$

$$\overline{P}_{1,1,1,0}(s) = \frac{\lambda_1\lambda_2\mu}{(s+\lambda_1+\lambda_2)(s+\lambda_1+\lambda_2+\mu)^2}\left[\frac{1}{s+\lambda_1+\lambda_2} + \frac{2p}{s+\lambda_1+\lambda_2+\mu}\right]$$

$$\overline{P}_{1,0,1,1}(s) = \frac{\lambda_1\lambda_2\mu}{(s+\lambda_1+\lambda_2)(s+\lambda_1+\lambda_2+\mu)^2}\left[\frac{1}{s+\lambda_1+\lambda_2} + \frac{2q}{s+\lambda_1+\lambda_2+\mu}\right]$$

$$\overline{P}_{1,1,1,1}(s) = \frac{2\lambda_1\lambda_2\mu^2}{(s+\lambda_1+\lambda_2)^2(s+\lambda_1+\lambda_2+\mu)^2}\left[\frac{1}{s+\lambda_1+\lambda_2} + \frac{1}{s+\lambda_1+\lambda_2+\mu}\right]$$

$$\overline{P}_{0,0,2,0}(s) = \frac{{\lambda_2}^2}{(s+\lambda_1+\lambda_2)(s+\lambda_1+\lambda_2+\mu)^2}$$

$$\overline{P}_{0,0,2,1}(s) = \frac{\lambda_2{}^2\mu}{(s+\lambda_1+\lambda_2)(s+\lambda_1+\lambda_2+\mu)^2}\left[\frac{1}{s+\lambda_1+\lambda_2} + \frac{1}{s+\lambda_1+\lambda_2+\mu}\right]$$

$$\overline{P}_{0,0,2,2}(s) = \frac{\lambda_2{}^2\mu^2}{(s+\lambda_1+\lambda_2)^2(s+\lambda_1+\lambda_2+\mu)^2}\left[\frac{1}{s+\lambda_1+\lambda_2} + \frac{1}{s+\lambda_1+\lambda_2+\mu}\right]$$

VERIFICATION OF THE MODEL

Taking Laplace Transform of $\overline{P}_n(t)$ given by $\overline{P}_n(s) = \int_0^\infty e^{-st} P_n(t)dt$ and from explicit results

$$\overline{P}_0(s) = \overline{P}_{0,0,0,0}(s) = \frac{1}{s+\lambda_1+\lambda_2}$$

$$\overline{P}_1(s) = \overline{P}_{1,0,0,0}(s) + \overline{P}_{1,1,0,0}(s) + \overline{P}_{0,0,1,0}(s) + \overline{P}_{0,0,1,1}(s) = \frac{\lambda_1+\lambda_2}{(s+\lambda_1+\lambda_2)^2}$$

$$\overline{P}_2(s) = \overline{P}_{2,0,0,0}(s) + \overline{P}_{2,1,0,0}(s) + \overline{P}_{2,2,0,0}(s) + \overline{P}_{1,0,1,0}(s) + \overline{P}_{1,1,1,0}(s) + \overline{P}_{1,0,1,1}(s) + \overline{P}_{1,1,1,1}(s) + \overline{P}_{0,0,2,0}(s) + \overline{P}_{0,0,2,1}(s) + \overline{P}_{0,0,2,2}(s) = \frac{(\lambda_1+\lambda_2)^2}{(s+\lambda_1+\lambda_2)^3}$$ and so on.

By taking Laplace inverse transform, we get

$$P_0(t) = e^{-(\lambda_1+\lambda_2)t}$$

$$P_1(t) = \frac{(\lambda_1+\lambda_2)t\, e^{-(\lambda_1+\lambda_2)t}}{1!}$$

$$P_2(t) = \frac{(\lambda_1+\lambda_2)^2\, t^2\, e^{-(\lambda_1+\lambda_2)t}}{2!}$$

and so on.

Therefore, $\sum_{n=0}^{\infty} P_n(t) = e^{-(\lambda_1+\lambda_2)t}\left\{1 + \frac{(\lambda_1+\lambda_2)t}{1!} + \frac{(\lambda_1+\lambda_2)^2t^2}{2!} + \cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots\right\}$

$$= e^{-(\lambda_1+\lambda_2)t}\, e^{(\lambda_1+\lambda_2)t} = 1$$

Hence, the verification.

This paper has analysed M/M/1 queue with non-preemptive priorities with a single server, Poisson arrivals of priority and non-priority units with different parameter. Service times are exponentially distributed. We have derived general equilibrium equations for *i* arrivals, *j* departures of priority units and *k* arrivals, *l* departures of non priority units.

Future work will focus on solving the system of equations by writing the algorithm for above queueing system and implementing this algorithm in MATLAB.

REFERENCES

[1] Balter M. H., Osogami T., Wolf A.S. and Wierman A., (2005) Multi-Server Queueing Systems with Multiple Priority Classes. *Queueing Systems: Theory and Applications.* **51** 331-360.

[2] Choi B.D., Kim B., and Chung J., (2001) M/M/1 Queue with Impatient Customers of Higher Priority. *Queueing Systems: Theory and Applications*. **38** 49-66.

[3] Cobham A.,(1954) Priority Assignment in Waiting Line Problems. *Operations Research.* **2** 70-76.

[4] Gaver D.P., (1962) A waiting line with interrupted service including priorities. *J. Roy. Stat. Soc. B.* **24** 073-90.

[5] Hawkes A.C., (1965) The time dependent solution of a priority queue with preemptive priorities. *Operations Research*. **13** 670-680.

[6] Heathcote C.R., (1959) The time dependent problem for a queue with preemptive priorities. *Operations Research.* **7** 670-680.

[7] Heathcote C.R., (1960) A Simple queue with several priority classes. *Operations Research.* **8** 631-638.

[8] Heathcote C.R., (1961) Preemptive Priority Queueing. *Biometerika*. **48** 57.

[9] Jaiswal N. K., (1961a) Preemptive resume priority queue. *Operations Research.* **9** 732-742.

[10] Jaiswal N. K., (1962) Time dependent solution of the head of the line priority queue. *Jr. Roy Statistic Soc. B*. **24** 73-90.

[11] Kao E. and Wilson S., (1999) Analysis of Non-preemptive Priority Queues with Multiple Servers and Two Priority Classes. *European Journal of Operational Research*. **118** 181-193.

[12] Miller R.G., (1960) Priority Queues. *Ann. Math. Stat*. **31** 86-103.

[13] Pegden C. D. and Rosenshine M., (1982) Some New Results for the M/M/1 queue. *Management Science*. **28** 821-828.

[14] Phipps T.E., (1956) Machine Repair as Priority Waiting Line Problem. *Operations Research*. **4** 76-85.

[15] Sharda, (1973a) On a Certain Type of Priority Queueing Problems. *Metrika*. **20** 93-100.

[16] Sharda, (1979) A Priority Queueing Problem with Intermittently Available Phase Type Service, *Cahiers du CERO*. **21**

[17] Sharda, (1981-1983) Preemptive Resume Priority Queueing Problem with Batch Arrivals and Departures and Intermittently Available Server. *Journal Of Mathematical Sciences*. **16-18** 117-134.

[18] White H. and Christie L.S., (1958) Queueing with Preemptive Priorities or with Breakdown. *Operations Research*. **6** 79-96.