Design and Application of Optimal Feedback Controller for controlling Active Vibrations

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ABSTRACT

Improved manufacturing methods have become crucial factors in retaining global competitiveness for a wide range of products. This has led to the development of intelligent, compact and lightweight smart machine tools. A great difficulty is faced in the design of such machine tools due to the fact that the reduction in weight results in low rigidity and poor vibration control characteristics. The induced vibrations in the system may destabilize the system and result in complete failure of the system. On the other hand, the successful vibration reduction of the smart machine tools during machining process can improve productivity, increase quality, and reduce tool wear. Also healing the vibration error can reduce industrial waste, save money and improve design flexibility of new cutting tools. Therefore, there is a need to develop machine tools that are equipped with suitable vibration control features. Objective of the present paper is an attempt to design an optimal feedback controller and its application to control the active vibrations. Smart machine tools technology using active control system may provide a solution to this problem. With the optimization of position of closed loop poles of the system, maximum reduction of vibrations has been found. These machine tools use piezoelectric materials, electro-rheological fluids and shape memory alloys as sensors and actuators for providing effective vibration control.

Keywords

Vibration control, Smart machine tools, optimal feedback controller and piezoelectric materials.

1. INTRODUCTION

It has been found that vibrations are produced in a machine tool due to cutting process itself rather when exciting forces come from outside elements. These vibrations are termed as self-induced vibrations or machine tool chatter. Till now, numerous methods have been devised and used to dampen out their impact, using cork slabs, rubber pads and steel springs etc. or by increasing the rigidity. Consequently the weight of machine tools increases which leads to high initial investment and more space utilization etc. A great difficulty is also faced in the design of such machine tools due to the fact that the reduction in weight results in low rigidity and poor vibration characteristics. Unless the vibration is effectively controlled, it may destabilize the system and may, very often, result in complete failure of the system. Therefore, there is need to develop machine tools that are equipped with suitable vibration control features. Smart machine tools technology using active control system may provide a solution to this problem [Meirovitch, 1986]. These machine tools use piezoelectric materials, electro-rheological fluids and shape memory alloys as sensors and actuators for providing effective vibration control.

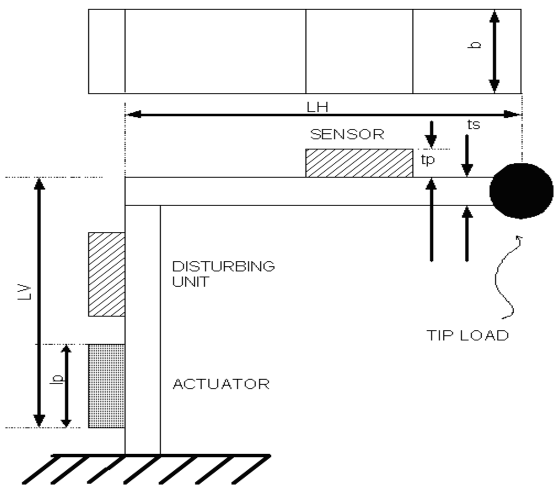
In this paper, a built-up structure made by joining two beams to form an inverted L – structure has been studied. Piezoelectric materials, bonded on a part of the surface of the structure, have been used as sensors and actuators. For controlling the vibrating structure using piezoelectric materials, the dynamics of these structures must be known. The relationship of electromechanical coupling of piezoelectric materials with the dynamics of these structures had been established by using finite element methods. Modal parameters, i.e. frequencies, damping ratios and mode shapes of inverted L – structure were calculated using the finite element model. Any flexible, vibrating system can be modeled into state space form using modal parameters.

Active vibration control is a feedback type of control. A feedback controller uses the measurements taken from the system to decide the inputs, which are given to control the system. The performance of the control system may be specified in terms of settling time, peak overshoot, steady state error and peak voltage required for actuators.

There are several techniques available, such as root locus method, bode plot technique, pole placement, state feedback, etc which can be used to design a controller satisfying these specifications. This forms the backbone of classical control theory. The principle tool in all these techniques is the Laplace Transformations. This approach makes extensive use of the concept of transfer functions. Pole placement method has been used for the controller design and finally the optimization has been done to finalize the position of closed loop poles to design optimal feedback controller.

2. DESCRIPTION OF STRUCTURE

In this paper, a built-up structure made by joining two beams to form an inverted L shaped structure has been studied. It is a simple extension of a one-dimensional beam to a two-dimensional beam structure. The practical applications of this structure are included in robots and machine tools. For example, SCARA robot and radial drilling machine is representative of an inverted L structure. This structure is also extensively used in flexible spacecrafts. Piezoelectric materials, bonded on a part of the surface of the structure, have been used as sensors and actuators. A Schematic View of the Inverted



L – Structure as shown in figure 1.

Fig. 1 Schematic View of the Inverted L – Structure

Table 1 Geometrical and Mechanical Properties

Dimension/Property	Material	
	STEEL	PZT
Length of Horizontal limb (mm)	L_H= 100	-------
Length of Vertical Limb (mm)	L_V=100	-------
Thickness (mm)	t_s=1	t_p=1
Length (mm)	l_s=20	l_p=20
Width (mm)	B=10	b=10
Young's Modulus (MPa)	E_s=210	E_p=64
Density (Kg/m^3)	ρ_s=7800	ρ_p=7650

Table 2 Electrical Properties of PZT

Property	Symbol	Value
Piezoelectric charge constant (m V^{-1})	d_{31}	171x10^{-12}
Piezoelectric charge constant (m V^{-1})	d_{32}	171x10^{-12}
Poisson's ratio	υ_p	0.28
Permittivity (Fm^{-1})	ε	106 x10^{-12}

3. MATHEMATICAL MODELING

Because of their coupled mechanical and electrical properties, piezoceramics have recently attracted significant attention for their potential application as sensors for monitoring and as actuators for controlling the response of structures. This new technology could possibly be applied to the design of large-scale space structures, aircraft structures, satellites, and so forth. Finite element analysis is the fundamental tool for modeling these flexible structures. A large amount of literature is available till date for mathematical modeling of these structures using finite element techniques. For controlling the vibrating structures using piezoelectric materials, the dynamics of these structures must be known. Electromechanical coupling of piezoelectric materials can easily be related with the dynamics of these structures using finite element methods.

Using the matrix iteration method [Meirovitch, 1986], the eigenvalue problem can be solved to give the natural frequencies and mode shapes of these structures. These parameters (i.e. natural frequencies and mode shapes) are the pre-requisites for designing an active vibration control system. Classical control systems are normally based on transfer functions. For multivariable systems with multi-inputs and multi-outputs, designer has to deal with matrices of transfer functions. In such cases, as the number of actuators and sensors are increased, the complexity of the control systems also increases. Thus for control of Multi-Input, Multi-Output systems, it is preferable to model the system in state-space form. Using the coupled control technique [2], the system can be written in state space form [3].

3.1 Lagrange's Equations of Motion for Linear Systems

The motion of a general linear system is given as [Meirovitch, 1986]

$$\sum_{i=1}^{n} \left[m_{ji} \ddot{\Delta}_i(t) + c_{ji} \dot{\Delta}_i + k_{ji} \Delta_i(t) \right] = Q_j(t) \qquad i, j = 1,2,\ldots\ldots, n \qquad (1)$$

Where Δ_i (t) is the physical displacement, $\dot{\Delta}_i$ (t) is physical velocity and $\ddot{\Delta}_i$ (t) is the acceleration at time instant t for the particular degree of freedom i. The vector of externally applied forces is denoted by $\mathbf{Q}_j$ (t). Also m, c and k are the elements of mass, damping and stiffness matrices respectively. Equation (1) represents a set of n simultaneous second – order ordinary differential equations in generalized coordinates, and are called Lagrange's equations of motion. This relation approximates infinitely many degree-of-freedom distributed systems, by an n-degree of freedom system. This relation can be written in matrix form as [Mohammed et. al., 1994]

$$\mathcal{M} \ddot{\Delta}(t) + \mathcal{C} \dot{\Delta}(t) + \mathcal{K} \Delta(t) = \mathcal{Q}(t) \qquad (2)$$

Where **M**, **C** and **K** are the global mass, damping and stiffness matrices respectively, and **Q** (t) is the vector of physical applied forces at various degrees of freedom on instant of time t. The column vector $\Delta(t)$ is the nodal (also called physical) displacements at time t. These global matrices are obtained from finite element modeling of structure.

3.2 Finite Element Modeling

Discrimination of the continuous L-structure into finite number of segments has been shown in Figure 2 .Any framed structure, vibrating in a single plane can be assumed to be made of 2D beam or frame elements [Chang, 1992]. Each of these elements may be loaded with axial as well as transverse forces, along with bending moments. At each axial end of the element, called a node, there are horizontal and vertical deflections and slopes. The stiffness and mass matrices for such element would be of the size 6x6 [Kumar et. al. 2005]. These 2D beam elements can be horizontal, vertical or at an angle of inclination 'α'. In case the element is mounted with a piezoelectric element, the mass and stiffness elements are modified [Kumar et. al. 2005]. Effective mass and flexural rigidity of L-Structure may be obtained by algebraically adding the masses and flexural rigidities of that of structure and attached PZT patches. The moment of Inertia (for flexural rigidity) of the element is taken about an axis perpendicular to the plane of the paper. Each element is having 6 degree of freedom as shown in figure 3.

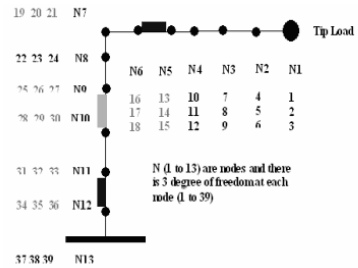


Fig.2 L-Structure (Discrete Form)

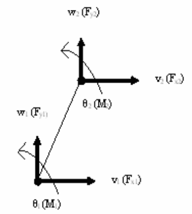


Fig.3 Six D.O.F. of 2D Beam Element

3.3 Natural Frequencies and Mode Shapes

The Lagrange's Equation (2), If C = 0 (undamped), Q (t) = 0 (without applying force) leads to following matrix eigenvalue - eigenvector problem for the natural frequencies w and their corresponding mode shape vector X.

$$M^{-1} KX = \omega^2 X \qquad (3)$$

The natural frequencies, the square roots of the eigenvalues of $M^{-1}K$ are obtained by

$$\det |K - \omega^2 M| = 0 \qquad (4)$$

It leads to *n*th order algebric equation in ω^2 with real coefficients to give systems *n* natural frequencies.

3.4 Modal Analysis

The eigenvalue problem associated with undamped free-vibration system for relation (2) is

$$(K - \omega^2 M)\,\phi = 0 \qquad (5)$$

where the transformation matrix ϕ is known as mass normalized Modal matrix.

Following transformations relates physical displacement 'Δ' to modal displacement 'q' at time instant t

$$\Delta(t) = \Phi\, q(t) \qquad (6)$$

This mode superposition method is used to transform the 'coupled equations of motion' in physical co-ordinates to a set of 'uncoupled equations of motion' in the modal co-ordinates

$$\Phi^T \mathcal{M} \Phi \ddot{q}(t) + \Phi^T \mathcal{C} \Phi \dot{q}(t) + \Phi^T \mathcal{K} \Phi q(t) = \Phi^T \mathcal{Q}(t) \quad (7)$$

where

$\Phi^T \mathcal{M} \Phi = I$,is unity matrix

$\Phi^T \mathcal{K} \Phi = \omega^2$ is a diagonal matrices containing the square of natural frequencies and

$\Phi^T \mathcal{C} \Phi = \tilde{C}$ is a symmetric damping matrix.

In case of proportional damping, the normalized damping matrix is given as [Rao and Gupta 1986]

$$\tilde{C} = \text{diag}[\ 2\ \xi_r\ \omega_r\ ] \quad (8)$$

where ξ_r is the damping ratio associated with r'th particular mode. Thus, the uncoupled system of equations with proportional damping takes the following form

$$\ddot{q}(t) + 2\ \xi\ \omega\ \dot{q}(t) + \omega^2 q(t) = \Phi^T Q(t) \quad (9)$$

the modal matrix Φ for an N degree of freedom system may be written in component form as

$$\begin{bmatrix} {}^{(1)}\Phi_1 & {}^{(2)}\Phi_1 & \cdots & {}^{(N)}\Phi_1 \\ {}^{(1)}\Phi_2 & {}^{(2)}\Phi_2 & \cdots & {}^{(N)}\Phi_2 \\ \vdots & \vdots & \ddots & \vdots \\ {}^{(1)}\Phi_N & {}^{(2)}\Phi_N & \cdots & {}^{(N)}\Phi_N \end{bmatrix} \quad (10)$$

Where ${}^{(j)}\Phi_k$ is the modal co-ordinate at k^{th} degree of freedom for j^{th} mode, and each column in the matrix represents the eigenvectors. The natural frequencies are collected to form the ω^2, diagonal matrix of modal frequencies squared, as

$$\omega^2 = \begin{bmatrix} \omega_1^2 & 0 & 0 & \cdots & 0 \\ 0 & \omega_2^2 & 0 & \cdots & 0 \\ 0 & 0 & \omega_3^2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \omega_N^2 \end{bmatrix} \quad (11)$$

where ω_i is the natural frequency of the i^{th} mode and i=1 to N.

For most of the structure systems under practical loading, only first few modes need to be considered. Thus the mode super position method is used to form a reduced order dynamic system [Kumar, 2007]. Depending upon the number of modes to be considered, the dimensions of the system are changed from N to R where R (R<N) is the reduced order model of the complete system.

3.5 Modal State Space Control

It has been mentioned earlier that for a physical structure only the first few modes are important from vibration control considerations. If such a structure is controlled by using discrete distributed actuators numbering 'a' and discrete distributed sensors numbering's', the following relations may be written.

$$\ddot{q}_k(t) + 2\xi_k \omega_k \dot{q}_k(t) + \omega_k^2 q_k(t) = \sum_{i=1}^{i=a} \left[{}^{(k)}\Phi_i \ \ Q_i(t) \right] \qquad k = 1,2,....r \quad (12)$$

For 'r' modes and 'a' actuators, equation (9) takes the form

where Q_i is the actual force acting at i^{th} degree of freedom and q_k is modal displacement at k^{th} mode. And the sensor output at the i^{th} degree of freedom, by the contribution of 'r' modes is given by

$$\Delta_i(t) = \sum_{k=1}^{k=r} \left({}^{(k)}\Phi_i \ \ q_k(t) \right) \qquad i = 1,2,....,s \quad (13)$$

where ${}^{(k)}\Phi_i$ is the mode shape, at i^{th} degree of freedom and for k^{th} mode.

Relation (12) for single actuator and single mode, in matrix form, is written as

where q(t), $\dot{q}(t)$ and $\ddot{q}(t)$ are the modal displacement, modal velocity and modal acceleration respectively. By making the substitution $w_1 = q(t)$ and $w_2 = \dot{q}(t)$ for the single particular mode, we may write the above equations as

$$\begin{Bmatrix} \dot{w}_1(t) \\ \dot{w}_2(t) \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & -2\xi\omega \end{bmatrix} \begin{Bmatrix} w_1(t) \\ w_2(t) \end{Bmatrix} + \begin{Bmatrix} 0 \\ \Phi_{actuator} \end{Bmatrix} \{Q_{actuator}\} \quad (14)$$

so that $w_1(t)$ is the modal displacement and $w_2(t)$ is the modal velocity. This type of representation is known as state space form. For the system having 'r' modes, equations (12) and (13) can be written in matrix state space form as

$$\dot{\mathcal{W}}(t) = \mathcal{F}\ \mathcal{W}(t) + \mathcal{G}\ u(t)$$
$$y(t) = \mathcal{H}\ \mathcal{W}(t) \quad (15)$$

where modal state vector is defined as

$$w(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \\ w_3(t) \\ w_4(t) \\ \vdots \\ w_{2r-1}(t) \\ w_{2r}(t) \end{bmatrix} \quad (16)$$

Such that $w_{2r-1} = q_r$, $w_{2r} = \dot{q}_r$. Other matrices i.e. **F**, **G** and **H** are called system matrices [Meirovitch, 1986].

3.6 Piezoelectric Sensing and Actuation

3.6.1 Piezoelectric Sensing

When a piezoelectric patch, attached to the distributed structure, is subjected to a change in slope at its two edges, electric charge is developed in the system. This charge developed in the PZT patch mounted on the steel structure is given by [8]

$$\delta(t) = \frac{1}{2}\left(t_s + t_p\right)\left(d_{31} + v_p\, d_{32}\right)\frac{E_p}{1-v_p^2}\, b\left(\theta_2(t) - \theta_1(t)\right) \quad (17)$$

Where $\theta_1(t)$ and $\theta_2(t)$ are slopes of end 1 and end 2, of PZT patch respectively, at t instant of time. The

$$\begin{Bmatrix}\dot{q}(t)\\ \ddot{q}(t)\end{Bmatrix} = \begin{bmatrix}0 & 1\\ -\omega^2 & -2\xi\omega\end{bmatrix}\begin{Bmatrix}q(t)\\ \dot{q}(t)\end{Bmatrix} + \begin{Bmatrix}0\\ \Phi_{actuator}\end{Bmatrix}\{Q_{actuator}\}$$

thicknesses of the steel structure and of the PZT patch are denoted by t_s and t_p respectively. The dielectric constants of the PZT patch are denoted by d_{31} and d_{32}. Widths of the beam as well as piezoelectric patch are represented by letter b. E_s and E_p denote respectively the Young's Modulus of Elasticity for the steel and piezoelectric material used. The values of these parameters are given in tables (1) and (2).

The voltage developed due to this charge is given by the relation [Butler, 1996]

$$\mathcal{V}(t) = \frac{\delta(t)t_p}{\varepsilon_p A_p} \quad (18)$$

Where A_p is the area of PZT patch ε_p is relative permeability.

Since all values except θ_1 (t) and θ_2 (t) are constant in equation (26), this equation may be written as

$$\mathcal{V}(t) = \Gamma\left(\theta_2(t) - \theta_1(t)\right) \quad (19)$$

Where Γ denotes a conversion coefficient. Its units are VC.

3.6.2 Piezoelectric Actuation

A PZT patch may be mounted over the vibrating structure to provide a controlling action. When a voltage 'V(t) ' volt is applied across the piezoelectric patch, at the instant of time t, the bending moment 'M_f (t)' of opposite sense is produced at both the edges. Value of this bending moment is given by [Baz and Poh, 1988]

$$M_f(t) = \left[\frac{d_{31}\ b\ E_p\langle E_s\ t_p\, t_s + E_s\ t_p^2\rangle}{2\langle E_p\ t_p + E_s\ t_s\rangle}\right] V(t) \quad (20)$$

Since all the parameters except V (t) are constant in relation (20), the following relation may be written

$$M_f(t) = \Psi\, V(t) \quad (21)$$

Where Ψ denotes the conversion coefficient. Its units are Nm/V.

4. System Transformations

4.1 Transformation from Continuous to Discrete Form

Where F and G and H are the system matrices in discrete form (whereas *F, G, H,* as found above, are system matrices in continuous form) and t is the sampling time. The state vector *W* is changed by W for the discrete system. In practice it is not necessary to solve these equations manually. There is a MATLAB function called **c2dm** that converts a given continuous system (either in transfer function or state-space form) to discrete system using the zero-order hold operation explained above. The basic command for this c2dm (in state space form) is

[F, G, H, J] = c2dm (*F, G, H, J*, Ts, 'zoh')

4.2 Transformation from State Space to Difference Equation Form

For the design of pole placement control the systems model in difference equation form is more suitable. Hence the state space model is converted into difference equation model. Using MATLAB command **ss2tf** the system in state – space form can be converted to transfer function form.

5. Feedback Controller Design

5.1 Finding Zeros (z), Poles (p) and Gain (k)

If the system is in state space form, parameters like Zeros (Z), Poles (P), and Gain (K) are obtained by following MATLAB command

[z, p, k] = ss2zp (A, B, C, D, iu)

- Zeros are the roots of numerator of the transfer function
- Poles the roots of denominator of the transfer function, and
- Controller gain is the ratio of the signal output to input of system (amplification)

5.2 Designing of Optimal Feedback Controller

It is observed [7] that as the imaginary part of the Closed Loop, CL poles were fixed and the real part of the CL poles is moved towards the y-axis, the performance of the system improves (i.e. better vibration attenuation). Similar performance was observed when the imaginary part of the CL poles was moved towards x-axis and the real part of the CL poles was fixed. In the later case, similar performance as compared to the former case was obtained at lesser control voltages as shown in Figure 4. So, the second approach was used in the present paper.

For making FB controller effective for forced vibrations, value of controller gains had to be increased. This can be done by iteratively solving the Diophantine equation with changed position of CL poles. During each iteration, CL poles are shifted towards x-axis on pole zero map. Any pole of the controller should not lie outside the unit circle disk on the complex plane.

By simulations, the performance comparison was made as a function of pole position of the controller transfer function as shown in Figure 5. Point 1 on Figure 5, corresponds to the very high gain controller with *characteristic equation corresponding to poles for 0g tip load*. The controller has poles with real part at 0, 0.6955 and -45. Clearly, it is an unstable controller. CL amplitude is more than Open Loop, OL amplitude in case of forced vibrations. *In case of free vibrations without any sensor noise, this is not the case. In that situation, the CL amplitude is less than OL amplitude*. But as real part of positive pole is moving towards unity, controller performance becomes better.

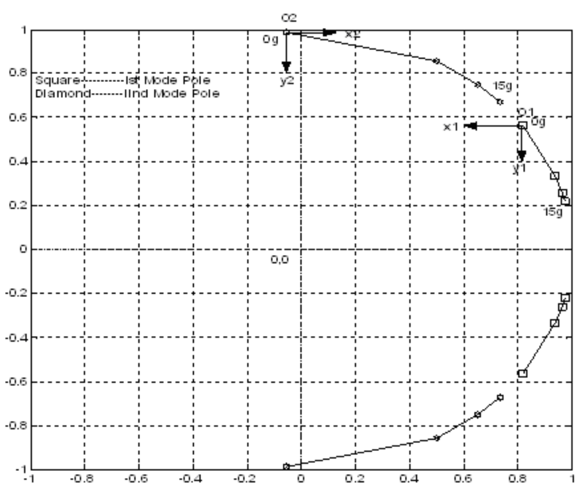


Fig. 4 Open Loop Poles of the System at Different Tip Loads

For points 2 to 5 on Figure 5**, closed loop poles are moved towards origin.** Instability starts from point 4. Where, the real parts of poles are 0.7774, **1.2785** and 0.7749. The real part of the largest positive pole is greater than unity. The FB controller corresponding to point 4 with poles (-0.7519, **1.1399** and 0.7226) on the Figure 5 is an unstable controller. However, the *closed loop system is stable and shows vibration reduction up to certain extent*. The optimum performance is obtained corresponding to point 3, for which all the controller poles are inside a unit circle disk. So, to maintain stability of controller, not even a single pole of the controller should lie outside the unit circle disk.

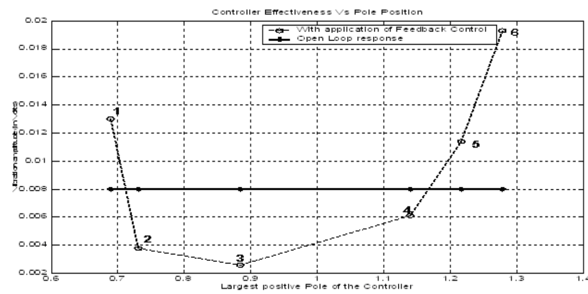


Fig. 5 Performance Comparisons Based on Pole Positioning of the Controller Transfer Function

Based on the above discussion the algorithm for controller design is summarized as below:

1. Input the system parameters and represent system in transfer function form using MATLAB command TF.
2. Find the OL poles of the transfer function of OL system using the MATLAB command POLE.
3. Find the magnitude of imaginary parts of the poles.
4. Reduce this magnitude by a small increment so that imaginary part of poles moves toward X-axis (but fix the real part).
5. Find CL pole, characteristic equation using the MATLAB command POLY using poles of step 4.
6. Find the controller parameters by solving the Diophantine equation [Wellstead et. al. 1988].
7. Find the transfer function of the controller using MATLAB command TF.
8. Find the poles of the controller using MATLAB command POLE.
9. Whether all the poles lie within the unit circle disk on imaginary plane?

10. If yes, go to 4, otherwise chose the previous controller (prior to the controller whose poles are outside the unit circle disk).

6. Conclusions

It is observed that as the imaginary part of the closed loop (CL) poles was fixed and the real part of the CL poles is moved towards the y-axis, the performance of the system improves (i.e. better vibration attenuation). Similar performance was observed when the imaginary part of the CL poles was moved towards x-axis and the real part of the CL poles was fixed. In the later case, similar performance as compared to the former case was obtained at lesser control voltages. Secondly, by optimizing the position of closed loop poles of the system, maximum reduction of vibrations is possible.

7. Future Trends

Present paper suggest about the optimization of position of closed loop poles of the system and maximum reduction of vibrations for free vibrations. So, the domain of systems imposed by forced vibrations can be explored.

8. Acknowledgements

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